The convex Lusternik-Schnirelmann category

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Abstract

We define and study what we call the *convex Lusternik-Schnirelmann category* of an open domain $D \subseteq \mathbb{R}^n$, or shortly $\operatorname{convcat}(D)$. These investigations are motivated by the observation that $\operatorname{convcat}(D)$ is an upper bound, sometimes sharp, for the number of solutions of the equation $f(x) = y$, where $f : D \to \mathbb{R}^n$ is an arbitrary CIP-function, namely a function which satisfies the *convex injectivity property*[a]. The main ingredients are the Zorn Lemma, the Herzog-Piranian global injectivity criteria, the Sard Theorem and the Hahn-Banach separation Theorem.

[a]A function $f : D \to \mathbb{R}^n$ is said to satisfy the *convex injectivity property*, or shortly f is a CIP-function, if its restrictions to the convex subsets of D are all injective