Possible convergence rates of order $\mathcal{O}\left(\frac{1}{n^2}\right)$ for some inertial algorithms obtained via the explicit Euler method applied to a perturbed version of the second order dynamical system that models Nesterov's accelerated convex gradient method

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Abstract. We deal with a general second order continuous dynamical system associated to a convex minimization problem. We show that inertial algorithms, such as Nesterov's algorithm, can be obtained via the natural explicit discretization from our dynamical system. Our dynamical system can be viewed as a perturbed version of the heavy ball method with vanishing damping, however the perturbation is made in the argument of the gradient of the objective function. This perturbation seems to have a smoothing effect for the energy error and eliminates the oscillations obtained for this error in the case of the heavy ball method with vanishing damping, as some numerical experiments show. We prove that the value of the objective function in a generated trajectory converges in order $O(1/t^2)$ to the global minimum of the objective function. Moreover, we obtain that a trajectory generated by the dynamical system converges to a minimum point of the objective function. Further, the inertial algorithms obtained via discretization from this system may have the same convergence rates and better convergence behaviour than Nesterov's accelerated convex gradient method.

References

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