Contents

Poster session 90

Preface

This volume contains the abstracts of the contributions to the 10th International Conference on Fixed Point Theory and Its Applications (ICFPTA-2012) held in Cluj-Napoca between July 9-18, 2012.

The purpose of the conference is to bring together leading experts and researchers in fixed point theory and to assess new developments, ideas and methods in this important and dynamic field. A special emphasis will be put on applications in related areas, as well as other sciences, such as the natural sciences, medicine, economics and engineering.

The conference continue a long and nice tradition of the previous fixed point theory meetings which were held in Marseille (1989), Halifax (1991), Seville (1995), Kazimierz Dolny (1997), Haifa (2001), Valencia (2003), Guanajuato (2005), Chiang Mai (2007) and Changhua (2009).

The program consists of three **Plenary talks** presented by Kazimierz Goebel, William Art Kirk and Ioan A. Rus, **Key Note talks, Short talks** and a **Poster Session**.

Finally, we would like to thank all the participants for their contribution to the success of this conference. We wish to all participants a fruitful conference and a nice stay in Cluj-Napoca and in Romania.

Adrian Petruşel
Radu Precup
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Plenary Lectures

Problems I left behind

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During over forty years of studying and working on problems of metric fixed point theory, I raised some problems and asked several questions. For some I was lucky to get answer or find followers who did it for me. Some are still open and seem to be difficult. Some are of my own and some came out after fruitful discussions with my friends and colleagues. The aim of this talk is to present a selection of them. The problems are devoted to: geometry of Banach spaces, minimal invariant sets, classification of Lipschitz mappings, stability of fixed point property, minimal displacement and constructions of optimal retractions.

Fixed point theorems in metric trees and arcwise connected topological spaces

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This will be a brief survey of certain topics in metric fixed point theory. Some new observations about fixed points in metric trees and in arcwise connected topological spaces will also be discussed.

Five open problems in the fixed point theory in terms of fixed point structures (I): Singlevalued operators

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Problem 1. *Problem of the invariant fixed point subsets.*

Let us have a fixed point theorem, T, and an operator f which does not satisfies the conditions of T. In which conditions the operator f has an invariant subset Y such that the restriction of f to Y, $f\big|_Y$, satisfies the conditions of T ?

In the terms of fixed point structures ($f.p.s.$) this problem takes the following form:

Let $(X, S(X), M)$ be a f.p.s. on X and $f : A \to A$ be an operator with $A \subset X$. In which conditions there exists $Y \subset A$ such that:

$$(a)\ Y \in S(X),\ (b)\ f(Y) \subset Y \text{ and } (c)\ f\big|_Y \in M(Y)\ ?$$

Problem 2. *Problem of the maximal f.p.s.*
Let $(X, S(X), M)$ be a f.p.s. and $S_1(X) \subset P(X)$ with $S_1(X) \supset S(X)$. Which are the f.p.s. with the following property:

$$S(X) = \big\{A \in S_1(X) \mid f \in M(A) \Rightarrow F_f \neq \emptyset\big\}\ ?$$

By definition, a solution of this problem is called a maximal $f.p.s.$ in $S_1(X)$. If $S_1(X) = P(X)$, then it is called a maximal $f.p.s.$

Problem 2$_a$. *Let $(X, S(X), M)$ be a maximal f.p.s. in $S_1(X)$. Does there exists $S_2(X) \supset S_1(X)$ such that $(X, S(X), M)$ is maximal in $S_2(X)$?*

Problem 2$_b$. *Let $(X, S(X), M)$ be a maximal f.p.s. in $S_1(X)$. Does there exists $M_1 \subset M$ such that $(X, S(X), M_1)$ is maximal in $S_1(X)$?*

Problem 3. *Problem of the pairs of operators with common fixed points.*
Let us have a fixed point theorem and f and g be two operators which satisfy the conditions of this theorem. In which conditions we have that $F_f \cap F_g \neq \emptyset$?
Which are the fixed point theorems T with the following property: If f and g satisfy the conditions of T, then $F_f \cap F_g \neq \emptyset$? Which are the fixed point theorems with the following property: If f and g satisfy the conditions of T and $f \circ g = g \circ f$, then $F_f \cap F_g \neq \emptyset$?

In the terms of $f.p.s.$ these problems take the following form:

Problem 3. *Let $(X, S(X), M)$ be a f.p.s., $Y \in S(X)$ and $f, g \in M(Y)$. In which conditions we have that, $F_f \cap F_g \neq \emptyset$?*

Problem 3$_a$. *Which are the f.p.s., $(X, S(X), M)$, with the following property:*

$$Y \in S(X),\ f, g \in M(Y)\ \Rightarrow\ F_f \cap F_g \neq \emptyset\ ?$$

Problem 3$_b$. *Which are the f.p.s., $(X, S(X), M)$, with the following property:*

$$Y \in S(X),\ f, g \in M(Y),\ f \circ g = g \circ f\ \Rightarrow\ F_f \cap F_g \neq \emptyset\ ?$$

By definition, a solution of this problem is called a $f.p.s.$ with the common fixed point property.

Problem 4. *Problem of the pairs of operators with coincidence points.*
Let us have a fixed point theorem and, f and g be two operators which satisfy the conditions of this theorem. In which conditions we have that, $C(f, g) \neq \emptyset$?

Which are the theorems T with the following property: If f and g satisfy the conditions of T and $f \circ g = g \circ f$, then $C(f,g) \neq \emptyset$?

In the terms of $f.p.s.$ these problems take the following form:

Problem 4. *Let $(X, S(X), M)$ be a f.p.s., $Y \in S(X)$ and $f, g \in M(Y)$. In which conditions we have that, $C(f,g) \neq \emptyset$?*

Problem 4_a. *Which are the f.p.s., $(X, S(X), M)$, with the following property:*

$$Y \in S(X),\ f,g \in M(Y),\ f \circ g = g \circ f \ \Rightarrow\ C(f,g) \neq \emptyset\ ?$$

By definition, a solution of this problem is called a $f.p.s$ with the coincidence property.

This problem includes: Horn's conjecture and Schauder-Browder-Nussbaum's conjecture.

Problem 4_b. *Which are the f.p.s, $(X, S(X), M)$, with the following property: For each $Y \in S(X)$ there exists $p_Y : Y \to Y$ such that*

$$f \in M(Y) \Rightarrow C(f, p_Y) \neq \emptyset\ ?$$

By definition, such an operator, p_Y, is called coincidence producing operator on $(X, S(X), M)$.

Problem 5. *Problem on the nonself operators.*

Let $(X, S(X), M)$ be a f.p.s., $Y \in S(X)$ and $f \in M(Y, X)$. In which conditions we have that $F_f \neq \emptyset$?

Problem 5_a. *For a given $f \in M(Y,X)$ to find an operator $\rho_f : Y \to Y$ such that:*

$$(a)\ \rho_f \in M(Y) \text{ and } (b)\ F_f = F_{\rho_f};$$

or

$$(a)\ F_{\rho_f} \neq \emptyset \text{ and } (b)\ F_f = F_{\rho_f}$$

We call such a ρ_f, a generalized retract of f.

Problem 5_b. *Which are the f.p.s., $(X, S(X), M)$ with the following property:*

For each $Y \in S(X)$ there exists a set retraction $\rho : X \to Y$ such that for all $f \in M(Y)$, $\rho \circ f \in M(Y)$?

In which conditions on f, we have that, $\rho \circ f \in M(Y)$ or $F_{\rho\circ f} \neq \emptyset$?

We call an f, for which $F_f = F_{\rho\circ f}$, retractible with respect to ρ and $\rho \circ f$ a retract of f.

Problem 5_c. *Let $f \in M(Y,X)$ and $\rho : X \to Y$ a set retraction. Which boundary conditions and which inwardness conditions imply that f is retractible with respect to ρ?*

Problem 5_d. *Conjecture of the generalized retracts.*

Each boundary condition and each inwardness condition on f implies the existence of a generalized retract of the nonself operator f.

Problem 5_e. *Let $(X, S(X), M)$ be a f.p.s., $Y \in S(X)$ and $f : Y \to X$ such that $Y \subset f(Y)$. In which conditions we have that $F_f \neq \emptyset$?*

Key Note Talks

Fixed point theoretic approach to the Collatz mapping

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The Collatz conjecture is a conjecture named after Lothar Collatz, who first proposed it in 1937, and so far, there exist several suggestive approaches to this conjecture, which are based on various mathematical research areas such as number theory, probability theory and computation theory.

In this talk, from the fixed point theoretic point of view, we investigate the asymptotic behavior of several dynamical systems which can represent the Collatz mapping as their subdynamical system.

Various applications of fixed point theorems in uniform spaces

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The main purpose of the paper is to review the various applications of fixed point theory to problems arising in the analysis of the transmission lines terminated by various configurations of nonlinear RGLC-loads. From mathematical point of view the investigation of the regimes for transmission lines leads to a mixed problem for a first order hyperbolic system. The nonlinearities in boundary conditions are caused by the type of characteristics of the nonlinear RGLC elements. In recent papers we have succeeded to reduce the mixed problems to initial value problems for functional differential equations of neutral type on the boundary. The nonlinearities in these equations are such as are the characteristics of RGLC elements. Of practical interest for these equations are periodic and oscillatory regimes. We would like to point out that the different circuits produce different systems of equations. We overcome the arising difficulties in each case by introducing suitable operators in function spaces of periodic (or oscillating) functions and prove an existence-uniqueness theorems as fixed points of the operator mentioned. Finally we note one more advantage of our method. We obtain successive approximations to the exact solution beginning with simple combinations of the trigonometric functions.

Fixed points, retractions, eigenvalues, and more

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It is well known that in every infinite dimensional Banach one can find the following pathological objects which are in sharp contrast to our geometric intuition:
(a) fixed point free continuous self-maps of the closed unit ball,
(b) retractions of the unit ball onto its boundary,
(c) homotopies joining the identity on the unit sphere to a constant map,
(d) nonzero maps without positive eigenvalues and normalized eigenvectors.

It is interesting to ask to what extent these maps may be chosen more regular than just continuous. For example, Lin and Sternfeld (1985) proved that the map in (a) (and therefore also in (b), (c), and (d)) may always be chosen Lipschitz continuous. In this talk we will address similar questions related to measures of noncompactness and condensing maps. It turns out that the answer is obvious for (a) and (d), but highly nontrivial for (b) and (c).

Fixed point theory for fractional equations

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If f and G are continuous then the fractional differential equation of Caputo type

$$^{c}D^{q}x = -G(t,x) + f(t), \quad x(0) = x_0, \quad 0 < q < 1, \tag{1}$$

can be inverted as $x(t) = x(0) + (1/\Gamma(q)) \int_0^t (t-s)^{q-1}[-G(s,x(s)) + f(s)]ds$ which, in turn, can be separated as the pair of integral equations

$$z(t) = x(0)\left[1 - \int_0^t R(s)ds\right], \tag{2}$$

$$x(t) = z(t) + (1/J)\int_0^t R(t-s)[g(s,x(s)) + f(s)]ds \tag{3}$$

where $J \geq 1$ is constant, $g(t,x) = Jx - G(t,x)$, and R is the completely monotone resolvent kernel with

$$0 < R(t), \quad \int_0^\infty R(s)ds = 1. \tag{4}$$

A myriad of real-world problems are described by (1), even for the single value $q = 1/2$. In [1], [2], and [3] we have shown that because of (4) Equation (3) can define a fixed point mapping suitable for a large array of fixed point theorems, notably of the Krasnoselskii, Schaefer, and Krasnoselskii-Schaefer type, using large contractions. Moreover, in *a priori* bound problems, the singularities in (1) and (3) coincide so that it is possible to construct Liapunov functionals for either (1) or (3), as shown in [4] and [5]. This talk will consist of a compact fixed point foundation for fractional differential equations.

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Methods for variational inequality problem over the intersection of fixed point sets of quasi-nonexpansive operators

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Many convex optimization problems in a Hilbert space $\mathcal{H}$ can be written as a variational inequality problem $VIP(\mathcal{F}, C)$ formulated as follows: Given a closed convex subset $C \subset \mathcal{H}$, find $\bar{u} \in C$ such that $\langle \mathcal{F}\bar{u}, z - \bar{u} \rangle \geq 0$ for all $z \in C$, where $\mathcal{F} : \mathcal{H} \to \mathcal{H}$ is strongly monotone and Lipschitz continuous. We consider a special case of $VIP(\mathcal{F}, C)$, where $C := \operatorname{Fix} T$ for a quasi-nonexpansive operator $T : \mathcal{H} \to \mathcal{H}$, i.e., an operator having a fixed point and satisfying $\|Tu - z\| \leq \|u - z\|$ for any $u \in \mathcal{H}$ and $z \in \operatorname{Fix} T$.

We present the following method for solving $VIP(\mathcal{F}, \operatorname{Fix} T)$: $u^{k+1} = T_k u^k - \lambda_k \mathcal{F} T_k u^k$, where $T_k : \mathcal{H} \to \mathcal{H}$, $k \geq 0$, are quasi-nonexpansive operators, $\bigcap_{k=0}^{\infty} \operatorname{Fix} T_k \supseteq \operatorname{Fix} T$ and $\operatorname{Fix} T_k$ approximate $\operatorname{Fix} T$ in some sense. We give sufficient conditions for the convergence of the method.

A convergence theorem for zeros of uniformly continuous generalized Phi-quasi accretive mappins

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Let E be a real Banach space and $A : E \to E$ be a uniformly continuous generalized Phi-accretive mapping. An iteration sequence is constructed which converges strongly to the unique solution of the equation $Au = 0$. This result is achieved by means of an incisive general result proved here that a uniformly continuous map is necessarily bounded.

On the means of projections on CAT(0) spaces

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We improve a result on approximation a common element of two closed convex subsets of a complete CAT(0) space appeared as Theorem 4.1 in [2]. New practical iterative scheme is presented and conditions on two given sets are relaxed.

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Looking for a renorming with the stable fixed point property

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A Banach space X is said to satisfy the *fixed point property (FPP)* if every non-expansive mapping T defined from a convex bounded closed subset C of X into C has a fixed point. We say that X satisfies the *stable FPP* if there exists a constant $d > 1$ such that every Banach space Y which is isomorphic to X satisfies the FPP whenever the Banach-Mazur distance between X and Y is less than d. Replacing the convex bounded closed set C by a convex weakly compact set we obtain the *weak fixed point property (wFPP)* and *the stable wFPP*. In this talk we will discuss whether it is possible or not to obtain a renorming with the stable FPP (or the stable wFPP) for several classes of Banach spaces. In particular, we will show that every separable space can be renormed to satisfy the stable wFPP but ℓ_∞ cannot. Furthermore the classic separable non-reflexive spaces c_0 and ℓ_1 cannot be renormed to satisfy the stable FPP. Our method is based upon the distortability (or non-distortability) of the space X in the sense of James' Distortion Theorem. In the final part, we will consider the possibility of using our method to prove the wFPP for some new classes of spaces.

The fixed point property for CAT(0) spaces for non convex and unbounded sets

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Let $T: C \to C$ a nonexpansive mapping and C a nonempty and closed subset of a Banach space. Then, in order to study the existence of fixed points of T, convexity and boundedness of C are usually required. To obtain results assuring the existence of fixed point for nonexpansive mappings in the lack of one of these two conditions is usually an extremely hard task. Certain results, however, have been obtained in this direction when both conditions are considered separately. Of a great interest is the case of Hilbert spaces, where obtained results have been especially relevant and deep. In this talk we address this kind of problems for geodesic CAT(0) spaces. In particular, we will present an extension of the result of W.O. Ray on the fixed point property of unbounded subsets of Hilbert spaces to the CAT(0) setting.

This talk is based on joint works with Bożena Piątek.

Semi-UKK spaces and fixed point property

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At this talk, we first introduce the class of semi-uniform Kadec–Klee spaces which is a uniform version of semi Kadec–Klee spaces studied by Vlasov. This class of spaces is a wider subclass of spaces with weak normal structure and hence generalizes many known results in the literature. We then give a characterization for a certain direct sum of Banach spaces to be semi-uniform Kadec–Klee and use this result to construct a semi-uniform Kadec–Klee space which is not uniform Kadec–Klee. At the end of the talk, we give a remark concerning the uniformly alternative convexity or smoothness introduced by Kadets et al.

This talk is based on the joint research by S. Saejung and J. Gao.

Existence and uniqueness for an evolution equation arising in growing cell population

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We prove that a nonlinear evolution equation which comes from a model of an age-structured cell population endowed with general reproduction laws is well-posed.

Some connections between renorming theory and fixed point property

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In this talk we will study new advances connecting renorming theory with fixed point theory for nonexpansive mappings.

Eigenvalue problems for nonlinear maximal monotone operators

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In this talk, we are concerned with nonlinear eigenvalue problem for pseudomonotone perturbations of maximal monotone operators in reflexive Banach spaces, motivated by the work of Kartsatos and Skrypnik [3]. Let X be an infinite-dimensional real reflexive Banach space such that X and its dual X^* are locally uniformly convex. Suppose that $T : D(T) \subset X \to 2^{X^*}$ is a maximal monotone multi-valued operator and $C : D(C) \subset X \to X^*$ is a generalized pseudomonotone quasibounded operator with $L \subset D(C)$, where L is a dense subspace of X. Applying a recent degree theory of Kartsatos and Skrypnik [2], we establish the existence of an eigensolution for the nonlinear inclusion $0 \in Tx + \lambda Cx$, in a regularization method by means of the duality operator. Moreover, possible branches of eigensolutions for the above inclusion are discussed. When the resolvents of T or C are compact, it was studied in [1, 3, 4], by applying the Leray-Schauder degree theory for compact operators. Furthermore, we give a surjectivity result about the operator $\lambda T + C$ when λ is not an eigenvalue for the pair (T, C), T being single-valued and densely defined.

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Logical extraction of effective bounds from proofs in nonlinear ergodic theory

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In this talk we give a survey on recent applications of the 'proof mining' program. This program is concerned with the extraction effective uniform bounds from ineffective proofs in analysis using techniques from proof theory. We will focus on problems in ergodic theory. More specifically, we discuss new explicit uniform rates of metastability (in the sense of Tao) for the von Neumann Mean Ergodic Theorem (for uniformly convex Banach spaces, [3]) as well as several nonlinear ergodic theorems due to Baillon and Wittmann (in the Hilbert space case, [1, 2]), Shioji/Takahashi (for Banach spaces with uniformly Gâteaux differentiable norm, [4]) and Saejung (for CAT(0)-spaces, [5]). The latter two results have been established in joint work with L. Leuştean using a novel method of eliminating Banach limits as well as a new logical metatheorem for uniformly smooth Banach spaces.

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Dynamics on Banach spaces with applications in life sciences

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An approach is presented in which large systems of interacting entities studied in life sciences, e.g., communities of biological cells, ecological systems, populations of plants and animals, are modeled as infinite interacting particle systems evolving in continuous space and time. Their evolution is described by means of differential equations in Banach spaces. Possible techniques for studying such equations, as well as the results obtained thereby, are also discussed.

Nash-type equilibria on Riemannian manifolds: a variational approach

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By using variational arguments, we provide a quite complete picture on the location, existence, uniqueness, and stability of Nash-type equilibrium points on Riemannian manifolds.In order to develop our results we combine various elements from non-smooth and variational analysis, dynamical systems and metric projections on Riemannian manifolds. I emphasize the strong curvature-dependence of the present study, asserting that the optimal geometric framework for Nash-type equilibria is the class of Hadamard manifolds. We also give some applications on the Poincare upper-plane model and on the cone of symmetric positive definite matrices endowed with the appropriate Killing form.

Perron-Frobenius and Krein-Rutman theorems for tangentially positive operators

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The famous Perron-Frobenius theorem asserts that a nonnegative matrix A admits a nonzero nonnegative eigenvector corresponding to the eigenvalue equal to the spectral radius of A. We shall discuss various extensions of these result in an infinite dimensional Banach space E endowed with the abstract ordering generated by a cone K in E including the well-known Krein-Rutman theorem for compact linear cone-invariant operators. The main issue is that the cone-invariance of an operator is replaced by suitable tangency properties. A typical result reads: if a bounded linear operator A on E is such that the essential spectral radius $r_{ess}(A) < s(A)$, where $s(A)$ stands for the spectral bound of A, and A is tangent to K, i.e., $Ax \in T_K(x)$ for all $x \in K$, where $T_K(x)$ is the tangent cone to the total cone $K \subset E$, then there is $x \in K$ such that $Ax = s(A)x$. This result generalizes previous results due to Nussbaum, Stuart and others. Moreover we shall discuss the availability of similar results concerning spectral properties of generators of strongly continuous semigroups of bounded linear

operators and, in particular, elliptic differential operators, including Arendt's characterization of positive semigroups on Banach lattices, Kato's inequality for differential operators, Phillips-Lumer theory of dispersive operators and others. Relation to the fixed point theory will be also remarked.

Fixed point theorems, convergence theorems and nonlinear ergodic theorems for new generalized nonlinear mappings in hilbert spaces with applications

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First, we introduce a new class of $(a_1, a_2, a_3, k_1, k_2)-$generalized hybrid mappings which is more general than the classes of nonexpansive mappings, nonspreading mappings, hybrid mappings, TY mappings, (α, β) generalized hybrid, mapping for $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$, λ-hybrid mappings for $\lambda > 1$, Kannan mapping, $k-$strictly pseudo-nonspreading mappings, contractive mapping, contractively nonspreading mapping, contractively hybrid mapping, and (α, β, γ) contractively hybrid mapping for $\alpha > 0$ in Hilbert spaces. Our new class of nonlinear mappings with nonempty fixed point sets may not be contained in the class of quasi-nonexpansive mappings. It this paper, we prove fixed point theorem, and two weak convergence theorems of Mann's type iteration process and Baillon's type iteration process for our class of mapping. Furthermore, using an idea of mean convergence, a strong convergence theorem of Halpern's type iteration process is proved for our class of mappings. We also apply our results to study fixed point theorems in Hilbert space of nonexpansive mapping, nonspreading mapping, hybrid mapping, TY mapping, λ-hybrid mapping, Kannan mapping, k-strict pseudo-nonspreading mapping, contractive mapping, contractively nonspreading mapping, contractively hybrid mapping, and (α, β, γ) contractively hybrid mapping. Our results will have many applications in fixed point theory. We give a unified approach to fixed point theorems and ergodic theorems and iterative processes of many classes of nonlinear mappings in Hilbert space.

Some relationships between sufficient conditions for the fixed point property

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The E-convex Banach spaces satisfy the so called Prus-Sczepanik condition. Moreover, we study the independence between this and several others sufficient conditions for the fixed point property of nonexpansive mappings in Banach spaces.

The role of nonexpansive type mappings in some optimization problems

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Thanks to the strong connection between nonexpansive mappings and monotone operators, it is posible to reformulate some algorithms to approximate a minimizer of a convex function as algorithms to approximate a fixed point of a nonexpansive type mapping. The goal of our talk is twofold: first we establish the foundations of such a connection and thus we show some research lines in metric fixed point which have been recently opened from this connection.

Iteration of averaging operators and Korovkin type theorems

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Korovkin's celebrated approximation theorem states that if a sequence of positive linear selfmaps $T_n : C([0,1]) \to C([0,1])$ converges to the identity on the subspace of at most second degree polynomials (i.e., $T_n x^k \to x^k$ for $k = 0,1,2$), then (T_n) converges to the identity on the entire space, that is, $T_n f \to f$ for all $f \in C([0,1])$.

One of our main result shows that if a sequence of positive linear selfmaps $T_n : C([0,1]) \to C([0,1])$ satisfies $T_n x^k \to x^k$ for $k = 0,1$ and $T_n x^2 \to x$ then $T_n f$ converges to the affine function given by $f(0)(1-x) + f(1)x$. The most important

particular case occurs when the sequence of linear operators (T_n) is constructed as the iteration sequence T_μ^n of the averaging operators T_μ defined by

$$(T_\mu f)(s) := \begin{cases} \int\limits_{[0,1]} f\left(\frac{st}{\mu_1}\right) d\mu(t) & \text{if } s \in [0, \mu_1], \\ \\ \int\limits_{[0,1]} f\left(\frac{s+t-st-\mu_1}{1-\mu_1}\right) d\mu(t) & \text{if } s \in [\mu_1, 1], \end{cases}$$

where μ is a Borel probability measure on $[0,1]$. It turns out that, provided that μ is not a Diac measure, the set of fixed points of T_μ consists of the 2-dimensional subspace of affine functions, and $T_\mu^n x^2 \to x$ which makes the above mentioned result appliable.

The main applications of averaging operators come form the theory of approximate convexity. As a consequence of the above Korovkin type result, starting from an approximate form of the Hermite–Hadamard inequality, we can deduce the relevant approximate convexity properties.

The results were jointly obtained with Judit Makó [1], [2].

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Some existence results for system of general variational-like inequality problems

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In this talk, we introduce a new type of system of generalized variational-like inequality problems for η-monotone operators in Banach spaces. We prove an existence theorem of solution for system of generalized variational-like inequality problems by using Fan-KKM theorem and the Kakutani-Fan-Glicksberg fixed point theorem. Our results in this paper extend and improve some known results in the literature.

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Fixed point methods in the study of nonlinear differential equations

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We are concerned with a class of quasilinear elliptic equations with gradient term and Dirichlet boundary condition. By means of related fixed point theorems and comparison principles, we establish various qualitative results.

A two-parameter control for contractive-like multivalued mappings

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We propose a general approach to defining a contractive-like multivalued mapping F which avoids any use of the Hausdorff distance between the sets $F(x)$ and $F(y)$. Various fixed point theorems are proved under a two-parameter control of the distance function $d_F(x) = dist(x, F(x))$ between a point $x \in X$ and the value $F(x) \subset X$. Here, both parameters are numerical functions. The first one $\alpha : [0, +\infty) \to [1, +\infty)$ controls the distance between x and some appropriate point $y \in F(x)$ in comparison with $d_F(x)$, whereas the second one $\beta : [0, +\infty) \to [0, 1)$ estimates $d_F(y)$ with respect to $d(x, y)$. It appears that the well harmonized relations between α and β are sufficient for the existence of fixed points of F. Our results generalize several known fixed point theorems.

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Some results and problems in fixed point theory

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In this lecture, I intend to offer an overview of some results of mine in fixed point theory and propose some related open problems. Here are two samples:

Theorem. *Let $(X, \langle\cdot,\cdot\rangle)$ be a real Hilbert space and let $T : X \to X$ be a nonexpansive potential operator.*

Then, the following alternative holds: either T has a fixed point, or, for each sphere S centered at 0, the restriction to S of the functional $x \to \int_0^1 \langle T(sx), x\rangle ds$ has a unique global maximum towards which each maximizing sequence in S converges.

Problem. Let X be a reflexive real Banach space, with unit sphere S, such that, for each compact function $f : S \to X^*$ satisfying

$$\inf_{x\in S} \|f(x)\|_{X^*} > 0$$

there exists $\hat{x} \in S$ such that

$$f(\hat{x})(\hat{x}) = \|f(\hat{x})\|_{X^*} \ .$$

Then, does X possess the Kadec-Klee property?

Mean recurrences

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Motivated by questions of algorithm analysis, we will discuss several approaches to determining convergence and limit values (fixed points) for recurrences of the form,

$$x_n := M(x_{n-m}, x_{n-m+1}, \cdots, x_{n-1}), \quad \text{for } n = m+1,\, m+2, \cdots,$$

where M is a (not necessarily linear) strict m-variable mean; that is,

$$\min(x_1, x_2, \cdots, x_m) \leq M(x_1, x_2, \cdots, x_m) \leq \max(x_1, x_2, \cdots, x_m),$$

with equality only when all variables are equal.

Conductivity imaging from minimal current density data

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Recently has been shown that the magnitude of one current density field suffices to recover the electrical conductivity inside a body. This interior data can currently be obtained from Magnetic Resonance data in the presence of direct/very low frequency current applied at the boundary of the object. One of the methods reduces the problem to minimizing a weighted gradient in the space of functions of bounded variation. In this talk I will present recent progress on this minimization problem. The results have been obtained jointly with A. Nachman and A. Moradifam.

Fixed point theorems and best proximity point theorems

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If the fixed point equation $Tx = x$ does not admit a solution, it is natural to search for a solution which is best suited. Such a point x, when it exists, is called a best proximity point. In this talk it is aimed to discuss the relation between Ky Fan type best approximation theorems, fixed point theorems and best proximity point theorems and recent results studied for the existence and convergence of iterative sequences to best proximity points. We will also indicate the status of some problems studied in the literature for the existence of best proximity points.

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Short Talks

Partially contractive type coupled fixed point theorems

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Recently, some authors have started to generalize fixed point theorems for contractive type mappings in a class of generalized type metric spaces in which the self distance need not be zero. These spaces, partial metric spaces, were first introduced by S. G. Matthews in 1994. The proved fixed point theorems have been obtained for mappings satisfying contractive type conditions empty of the self distance. In this article we prove some coupled fixed point theorems for mappings satisfying contractive conditions allowing the appearance of self distance terms. These partial contractive type mappings do reflect the structure of the partial metric space and hence their coupled fixed theorems generalize the previously obtained by H. Aydi in " Some coupled fixed point results on partial metric spaces, International Journal of Mathematical Sciences, Article ID 647091, 11 pages (2011)". Some examples are given to support our claims.

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Fixed point theorems for mappings satisfying general contractive condition of integral type in G-metric spaces

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In this paper, we prove some theorems on fixed and common fixed points for mappings satisfying general contractive condition of integral type in a complete G-metric space. Our results are extensions of the results of Debashis Dey, Anamika Ganguly and Mantu Saha [2] and generalizations of several results in the literature including the results of Branciari [1].

Kernel perturbation of Picard iterates for first order nonlinear systems with nonlocal initial conditions

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The aim of this paper is to construct approximations for the solutions of the following first order differential system with nonlocal initial conditions using kernel perturbations of the Picard iterates:

$$\begin{cases} x'(t) = f_1(t, x(t), y(t)) \\ y'(t) = f_2(t, x(t), y(t)), \text{ a.e. on J} \\ x(0) = \alpha[x] \\ y(0) = \beta[y]. \end{cases} \tag{1}$$

The existence and uniqueness of the solutions in $W^{1,1}(0,1)$ was studied in [9], [10] using vectorial norms and convergent to zero matrices. We use the same techniques and conditions in order to construct numerical approximations of the solutions in $W^{1,1}(0,1)$ and in $W^{1,2}(0,1)$ using iterates of the form

$$\begin{cases} x_{n+1}(t) = \frac{1}{1-\alpha[1]}\alpha[g_{1,n}] + \int_0^t P_n(f_1(s, x_n(s), y_n(s)))ds \\ y_{n+1}(t) = \frac{1}{1-\beta[1]}\beta[g_{2,n}] + \int_0^t P_n(f_2(s, x_n(s), y_n(s)))ds \end{cases}, \ \forall t \in [0, k],$$

where

$$g_{1,n}(t) = \int_0^t P_n(f_1(s, x_n(s), y_n(s)))ds,$$

$$g_{2,n}(t) = \int_0^t P_n(f_2(s, x_n(s), y_n(s)))ds, \ \forall t \in [0,1]$$

and P_n is a linear, positive and uniform operator (Bernstein, Nevai, truncated Chebyshev series). Besides the theoretical convergence results we also give comparative numerical results for a set of test problems.

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Fixed point and mean convergence theorems for hybrid mappings

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We first introduce the class of hybrid mappings in Hilbert spaces. This class contains the classes of nonexpansive mappings and nonspreading mappings in Hilbert spaces. Then we show a fixed point theorem and a mean convergence theorem for such mappings.

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Firmly nonexpansive mappings in geodesic metric spaces

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In this short talk we firstly introduce the concept of firmly nonexpansive mappings in the setting of W-hyperbolic spaces. We prove the existence, under mild conditions, of periodic points and fixed points for nonexpansive and firmly nonexpansive mappings in uniformly convex W-hyperbolic spaces. Some of these results unify and strength previous ones. We also give a proof of the Δ-convergence to a fixed point of Picard iterates for firmly nonexpansive mappings, which is obtained from the asymptotic regularity of this class of iterates. We get an effective rate of asymptotic regularity for firmly nonexpansive mappings (this result is new, as far as we know, even in linear spaces). Finally, we apply our results to a minimization problem. More precisely, we prove the Δ-convergence of a proximal point like algorithm to a minimizer of a convex proper lower semi-continuous function defined on a CAT(0) space.

Every result I will show can be found in [1], which is a joint work with L. Leuştean and G. López.

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Fixed point and common fixed point of mappings on CAT(0) spaces

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In this talk, we discuss with on two mappings T and S, where T and S belongs to the class of mappings which satisfying in conditions (E) and (C_λ) as generalization of Suzuki's condition (C), which define on a CAT(0) space X, also we shall verify the existence of a common fixed point for these two mappings. Our result improves a number of very recent results.

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Existence criteria for the solutions of three types of variational relation problems

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Variational relation problems were introduced by Luc in [1] as a general model for a large class of problems in nonlinear analysis and applied mathematics. Since this manner of approach provides unified results for several mathematical problems it has been used in many recent papers (see [2-9]). In this paper we investigate the existence of solutions for three types of variational relation problems which encompass several generalized equilibrium problems, variational inequalities and variational inclusions studied in a long list of papers in the field.

Assume that X is a convex set in a topological vector space and Y and Z are two sets, endowed for each problem with an adequate topological and/or algebraic structure. Let $T : X \multimap Y$, $P : X \multimap Z$ be two set-valued mappings and $R(x, y, z)$ be a relation linking elements $x \in X$, $y \in Y$, $z \in Z$. The investigated problems are the following:

(VRP 1_a) Find $\bar{x} \in X$ such that $R(\bar{x}, y, z)$ holds for all $y \in T(\bar{x})$ and all $z \in P(\bar{x})$.

(VRP 1_b) Find $\bar{x} \in X$ such that for each $y \in T(\bar{x})$ there exists $z \in P(\bar{x})$ such that $R(\bar{x}, y, z)$ holds.

(VRP 2) Find $\bar{x} \in X$ and $\bar{z} \in P(\bar{x})$ such that $R(\bar{x}, y, \bar{z})$ holds for all $y \in T(\bar{x})$.

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On the strong and $\triangle$-convergence of S-iteration process for generalized nonexpansive mappings on CAT(0) space

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In this paper, we give the strong and $\triangle$-convergence theorems for the S-iteration process of generalized nonexpansive mappings on CAT(0) space which extend and improve many results in the literature.

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On some fixed point theorems for contractive type mappings defined on product spaces

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We survey some recent fixed point results established by the first author ([3]-[7]) for mappings $f : X^k \to X$ satisfying Prešic type contractive conditions and introduce and study the stability of a multi step fixed point iterative method, in the case (X, d) is a complete metric space, k a positive integer, and there exist a constant $a \in \mathbb{R}$ such that $0 < ak(k + 1) < 1$ and $f : X^k \to X$ is a mapping satisfying the following contractive type condition (introduced and studied in [6]):

$$d(f(x_0, \dots, x_{k-1}), f(x_1, \dots, x_k)) \leq a\sum_{i=0}^{k} d(x_i, f(x_i, \dots, x_i)), \tag{PK}$$

for any $x_0, x_1, \dots, x_k \in X$.

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N dimensional Riesz angle

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In [1] Borwein and Sims defined the Riesz angle as follow

$$\alpha(X) = \sup\{\||x| \vee |y|\| : x, y \in B(X)\}.$$

They proved that a Banach space X has the weak fixed point property for nonexpansive mappings if there exists a weakly orthogonal Banach lattice Y such that $d(X,Y)\alpha(Y) < 2$. We define N dimensional Riesz angle by

$$\alpha_N(X) = \sup\left\{\Big\| \bigwedge_{\substack{i,j=1,2,\ldots,N \\ i\neq j}} (|x_i| \vee |x_j|)\Big\| : x_1, x_2, \ldots, x_N \in B(X)\right\}.$$

The properties of N dimensional Riesz angle and the fixed point theorem that generalizes Borwein and Sims's result will be presented.

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Extending the method of successive approximations for integral equations

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Two extensions of the method of successive approximations for integral equations are considered. On the one hand, a complete numerical approach is realized for integral equations with deviating argument, combining the technique of successive approximations with an optimal interpolation procedure. On the other hand, the method of successive approximations is extended at integral equations for which the solution take values in a metrical monoid.

Mann iteration process for asymptotic pointwise nonexpansive mappings in metric spaces

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Let (M, d) be a complete 2-uniformly convex metric space. Let C be a nonempty, bounded, closed, and convex subset of M and $T : C \to C$ be an asymptotic pointwise nonexpansive mapping. In this talk, we prove that the modified Mann iteration process defined by

$$x_{n+1} = t_n T^n(x_n) \oplus (1 - t_n)x_n$$

converges in a weaker-sense to a fixed point of T.

Minimal displacement and fixed point property for lipschitzian and uniformly lipschitzian mappings

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We show recent results concerning the minimal displacement problem for lipschitzian and uniformly lipschitzian mappings. We also show examples concerning stability of the fixed point property for uniformly lipschitzian mappings.

The Denjoy-Wolff theorem for condensing and holomorphic mappings

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Using the Kobayashi distance k_D, we establish the Denjoy-Wolff theorem for condensing and holomorphic self-mappings of a bounded and strictly convex domain in a complex Banach space.

A second order analysis of the periodic solutions for nonlinear periodic differential systems with a small parameter

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We deal with nonlinear T–periodic differential systems depending on a small parameter. The unperturbed system has an invariant manifold of periodic solutions. We provide the expressions of the bifurcation functions up to second order in the small

parameter in order that their simple zeros are initial values of the periodic solutions that persist after the perturbation. In the end two applications are done. The key tool for proving the main result is the Lyapunov–Schmidt reduction method applied to the T–Poincaré–Andronov mapping. The results presented here are extensions to more general cases of the results from [1].

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Fixed point theorems for the Hyers-Ulam stability of functional equations

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The fixed point method is an important technique used for proving the Hyers-Ulam stability of functional equations. The goal of this talk is to present applications of the fixed point theorems of Banach, Diaz & Margolis, Luxemburg-Jung, Bianchini-Grandolfi and Matkowski-Rus, to the theory of Hyers-Ulam stability of some functional equations.

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Existence results for integral inclusions via fixed points

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The study of existence of solutions for several boundary value problems associated to differential inclusions of the form $\mathcal{D}x \in F(t,x)$, where $\mathcal{D}$ is a differential operator and $F(.,.) : I \times X \to \mathcal{P}(X)$ is a set-valued map reduces often to the existence of solutions for Fredholm-type integral inclusions of the form

$$x(t) = \lambda(t) + \int_0^T G(t,s)u(s)ds, \quad u(t) \in F(t,x(t)) \quad a.e.\ (I), \tag{1}$$

where $I = [0,T]$, $\lambda(.) : I \to X$, $F(.,.) : I \times X \to \mathcal{P}(X)$, $G(.,.) : I \times I \to \mathbf{R}$ are given mappings and X is a separable Banach space.

We provide several existence results for problem (1), when the set-valued map $F(.,.)$ has convex or nonconvex values. This results are essentially based on a nonlinear alternative of Leray-Schauder type, Bressan-Colombo selection theorem for lower semicontinuous set-valued maps with decomposable values and Covitz and Nadler set-valued contraction principle.

Non-compact equilibrium points and applications

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We prove an equilibrium existence result for vector functions defined on noncompact domain and we give some applications in optimization and Nash equilibrium for noncooperative game.

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Generalized contractions in metric spaces endowed with a graph

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A very interesting approach, in the theory of fixed points, was recently given by J. Jachymski [4] and G. Gwóźdź-Lukawska, J. Jachymski [3], by using the context of metric spaces endowed with a graph. They get an extension of the classical Banach contraction principle in this setting.

The purpose of this article is to present some new fixed point results for graphic contractions and for Ćirić-Reich-Rus G-contractions on complete metric spaces endowed with a graph. The particular case of almost contractions is also considered.

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Fixed point results for contractive type mappings on cone metric spaces involved with a graph

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We prove some fixed point theorems for contractive type mappings on complete cone metric spaces associated with a graph. The sufficient conditions for the existence of such contractions are obtained. The investigated results also extend some others existed in the current literature.

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Compactness results for differentiable functions and applications to the boundary value problems via fixed point methods

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Compactness criteria are useful tools in fixed points theory. In this talk we characterize the relative compactness of subsets of the space $BC^m([0,+\infty[;E)$ of bounded and m-differentiable functions defined on $[0,+\infty[$ with values in a Banach space E. Moreover, we apply this characterization to prove the existence of solutions of a boundary value problem in Banach spaces.

Ekeland Variational Principle in asymmetric locally convex spaces and in quasi-uniform spaces

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Ivar Ekeland proved in 1972 ([4]) the existence of a minimum point for a perturbation of a lower bounded lsc extended real-valued function defined on a complete metric space. This result, known as Ekeland Variational Principle (EkVP), turned out to be a powerful and versatile tool in many branches of mathematics – Banach space geometry, optimization, economics, etc. (see, for instance, [4] and [6]).

In the paper [1] it was proved a variant of EkVP in quasi-metric spaces satisfying some completeness conditions, with applications to fixed point theorems of Caristi and Clarke type. In the present talk (based on [2]) we shall discuss some variants of EkVP in asymmetric locally convex spaces and in quasi-uniform spaces, extending to the asymmetric case some results obtained by Hammel [7, 8] and Qiu [9].

The lack of symmetry in the definition of quasi-metric and quasi-uniform spaces causes a lot of troubles, mainly concerning completeness, compactness and total boundedness in such spaces. A thorough presentation of quasi-metric, quasi-uniform, asymmetric normed and asymmetric locally convex spaces is given in the forthcoming book [3].

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Coupled fixed point results in symmetric G-metric space

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Mustafa et. al [14,15] generalized the concept of metric space by introducing G-Metric Space and proved fixed point theorems for mappings satisfying different contractive conditions [see 16-20]. In this article, we introduce the notion of w-compatible maps, b-coupled coincidence points and b- common coupled fixed points for non self maps and obtain fixed point results using these new notions in symmetric G-Metric Space. It is worth mentioning that our results neither rely on completeness of space nor the continuity of any maps involved therein. Also relevant examples have been cited to illustrate the existence of solution of system of linear integral equations. Our work sets analogues, unifies, generalizes, extends and improves several well known results existing in literature, in particular the recent results of B. Fisher, B. Singh and S. Jain, C.T. Aage and J.N.Salunke, G. Jungck, M. Gugani, M. Aggarwal and R. Chugh, W. Shantawi.

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Approximation of fixed points of continuous bounded pseudo-contractive mappings

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An iteration process is proved to converge *strongly* to a fixed point of T where T is a bounded continuous and pseudo-contractive mapping on certain real Banach spaces. The ideas of the iteration process are applied to approximate zeros of continuous bounded and accretive maps. Finally, application to a convex minimization problem is given.

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Best proximity points for non-self mappings

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A classical and very well-studied problem in Metric Fixed Point Theory is the existence of fixed point of single-valued non-self mappings $T : A \to B$, where A and B are subsets of a metric space X. Attending to the nature of these mappings, the theory has not only studied the classical fixed point problem that tries to find solutions of the equation $Tx = x$ but also has considered those situations where the existence of fixed points may even make no sense. In these special cases, the interest mainly focuses on the existence of approximate solutions of the equation $Tx = x$, that is, points that are close to its image somehow. In this direction, one classical well-known best approximation theorem due to Fan [1] is the one that states that if A is a compact, convex and nonempty subset of a Hausdorff locally convex topological vector space X and $T : A \to X$ is a continuous mapping, then there exists a point $x \in A$ such that $d(x, Tx) = d(Tx, A)$. Different notions of approximate solutions have been considered in the literature. As a consequence, many new fixed point problems have arisen. Our interest here mainly focuses on the existence of approximate solutions of the equation $Tx = x$ which in turn are solutions of the equation $d(x, Tx) = inf\{d(x, y) : x \in A,\ y \in B\}$. Points that satisfy the previous equality are known in the theory either as absolute optimal approximate solutions or as best proximity points.

In this lecture, we improve some recent results of the theory concerning the so-called proximal contractions [2]. On the other hand, we study some geometrical properties relative to pair of sets that guarantee the existence of best proximity points in different contexts.

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A product space with the fixed point property

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I will talk about a joint work with Enrique Llorens Fuster, showing that the l_1 sum of the van Dulst space with itself has the fixed point property, although this does not follow directly from the known results of l_1 sums of spaces with the (FPP).

A unified approach to iterative construction of common fixed points in nonlinear domains

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W. Takahashi [Kodai Math. Sem. Rep., 22 (1970), 142-149] introduced the notion of convexity in a metric space. Subsequently, many authors have studied fixed point theory in convex metric spaces. Recently, U. Kohlenbach [Trans. Amer. Math. Soc., 357 (2005), 89-128] has enriched convex metric space as hyperbolic space by including some additional conditions. The characterization of a uniformly convex Banach space in terms of an inequality by H. K. Xu [Nonlinear Anal. 16 (1991), 1127–1138, Theorem 2] is playing a pivotal role in the development of the iterative construction of fixed points on linear domains. Y. X. Tian [Comput. Math. Appl. 49(2005), 1905-1912] and I. Yildrim and S. H. Khan [Appl. Math. Comput. 218 (2012), 4860-4866] have considered modified Takahashi's convex structure to study certain iterative algorithms in a convex metric space. We note that neither the modified convex structure contains Takahashi's convex structure as a special case nor it is suitable to generalize the above mentioned characterization of Xu to metric spaces.

W. A. Kirk and B. Panyanak [Nonlinear Anal. 68(2008), 3689-3696] have introduced and studied the concept of $\triangle-$convergence in the special class of metric spaces known as $CAT(0)$ spaces. In the Banach space setting, it coincides with the usual weak convergence.

In this talk, we propose an implicit algorithm for two finite families of asymptotically nonexpansive maps due to K. Goeble and W. Kirk [Proc. Amer. Math. Soc. 35 (1972), 171-174] in a hyperbolic space and present $\triangle-$convergence and strong convergence results for the proposed algorithm. Our convergence results refine and generalize several recent results in uniformly convex Banach spaces and $CAT(0)$ spaces. It is worth mentioning that Y. X. Tian and I. Yildrim and S. H. Khan have used modified convex structure and so they were unable to extend their results to uniformly convex metric spaces. We employ the classical definition of convexity due to Takahashi in our work and hence the corresponding known results in uniformly convex Banach spaces as well as $CAT(0)$ spaces can be deduced simultaneously from our results.

The fixed point property in the space c_0 with an equivalent norm

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We study the fixed point property (FPP) in the Banach space c_0 with the equivalent norm $\|\cdot\|_D$. The space c_0 with this norm has the weak fixed point property. We prove that every infinite dimensional subspace of $(c_0, \|\cdot\|_D)$ contains a complemented asymptotically isometric copy of c_0, and thus does not have the FPP, but there exist nonempty closed convex and bounded subsets of $(c_0, \|\cdot\|_D)$ which are not ω-compact and do not contain asymptotically isometric c_0-summing basis sequences. Then we define a family of sequences which are asymptotically isometric to different bases equivalent to the summing basis in the space $(c_0, \|\cdot\|_D)$ and we give some of its properties.

Linear horseshoes as random fixed points in affine IFS

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We are concerned with set-valued dynamics without contractiveness (the dynamics of contracting multi-functions is quite well studied, especially while looking for fixed points, see, e.g., I. A. Rus, A. Petruşel, G. Petruşel, Fixed point theory. Cluj University Press, Cluj-Napoca, 2008). Some attempts for non contractive case have been done by A. Avila, J. Bochi and J.-C. Yoccoz (2008), who considered uniformly hyperbolic *SL(2,R)*-cocycles, V. Glavan and V. Guţu (2009),who studied affine IFS with shadowing property, and V. Bergelson, M. Misiurewicz and S. Senti (2006), who studied the dynamics of a semi-group, generated by affine functions $\{1/2x, 3x+1\}$, and in connection with the famous and yet unsolved $(3n+1)$-problem.

We consider an affine IFS of the type $x \mapsto Ax + b_i$ with diagonal 2x2 matrix $A = (\lambda, \mu)$, where $|\mu| < 1 < |\lambda|$ and $b_1 = (0,0), b_2 = (1-\lambda, 0), b_3 = (0, 1-\mu), b_4 = (1-\lambda, 1-\mu)$.

For most of the points $x \in R^2$ all bi-infinite chains, as combinations of these affine mappings, are unbounded, except at least for fixed points. But there are other points that generate bounded bi-infinite chains. We englobe them in the set under the notation Λ, and this set will play the role of the "linear horseshoe" in what follows. We characterize Λ as a closed and viable on Z subset, i.e., we prove that $x \in \Lambda$, if and only if there is a bi-infinite chain, that starts at x, and which lies for all time in Λ. It is worth noting, that viability is much weaker than invariance, but it looks quite naturally in context of multi-valued dynamics. Depending on the parameters λ and μ, Λ may be a Cantor set, as the classical horseshoe, or a direct product of a Cantor set and a closed interval, or the whole square with fixed points as edges.

Another characterization of this set we give in terms of random dynamical systems, or in other words, in terms of skew product flows over the Bernoulli shift. We prove that the affine IFS generates a suitable affine skew-product flow over the Bernoulli shift, and this flow, in turn, has a continuous invariant cross-section of the fiber-bundle $\Sigma_4 \times R^2 \to \Sigma_4$. This invariant cross-section is called also as random fixed point, mostly in random dynamical systems. We prove that Λ coincides with the horizontal projection on the fiber of this invariant cross-section. We also characterize the invariant cross-section of the affine skew-product flow as the genuine fixed point of a suitable constructed mapping on the space of all continuous cross-sections, and this fixed point is unique.

Common fixed point theorems for pairs of subcompatible and subsequentially continuous maps

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In this paper, we introduce the new concepts of subcompatibility and subsequential continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four maps in a metric space which improves a recent result of Jungck and Rhoades [3]. Also we give another common fixed point theorem for two pairs of subcompatible maps of Greguš type which extends results of the same authors, Djoudi and Nisse [2], Pathak et al. [5] and others and we end our work by giving a third result which generalizes results of Mbarki [4] and others.

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On the connectedness of attractors of affine hyperbolic IFS

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S. B. Nadler Jr. [1] and J. Hatchinson [2] have shown that any hyperbolic Iterated Function System (IFS), consisting of a finite collection of contractions in a complete metric space X, possesses a unique invariant compact set, called the attractor of this IFS.

Namely, if $\{X; f_1, \dots, f_m\}$ is an IFS, consisting of contractions $f_i : X \to X$ with Lipschitz constants L_i, then the attractor is the fixed point of the respective Nadler-Hutchinson operator and the unique solution of the equation $\bigcup\limits_{i=1}^{m} f_i[A] = A$, where $f_i[A] = \{f_i(x) \,|\, x \in A\}$.

The structure of attractor, in particular, its connectedness, was studied by many authors. R. Williams [3] has shown that the attractor is totally disconnected provided $\sum\limits_{i=1}^{m} L_i < 1$. A. A. Ordin and Ya. V. Kucherinenko [4] have established some sufficient conditions for the attractor of an affine IFS, consisting of affine contractions in the Euclidean space, to be totally disconnected.

We will present some generalizations of these results.

Theorem. *Let $\{\mathbb{R}^n; f_1, \dots, f_m\}$ be an affine hyperbolic IFS, consisting of nonsingular affine contractions f_i in $\mathbb{R}^n$ and let A denote its attractor.*

1. *If the attractor A is totally disconnected, then $\sum\limits_{i=1}^{m} |\det f_i| < 1$, where $\det f_i$ denotes the determinant of the linear part of f_i.*
2. *If $\min\limits_i |\det f_i| + \max\limits_i |\det f_i| \geq 1$, then the attractor A is connected.*

We will discuss also the problem of representation of compact sets as attractor of a hyperbolic IFS.

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Shrinking projection methods for a family of generalized nonexpansive mappings in a Banach space

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In this talk, motivated by the result of Kimura and Takahashi [2] and that of Plubtieng and Ungchittrakool, [3] we prove a strong convergence theorem for finding a common fixed point of generalized nonexpansive mappings in Banach spaces by using the shrinking projection method.

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An approximation of the solution of some variational inequalities

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to be announced

Some remarks on demiclosedness principle

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Suppose X is uniformly convex, Y is a Banach space and $\|\cdot\|_Z$ is a norm in $X \oplus Y$ that satisfies certain conditions. Let C be a nonempty, convex and weakly compact subset of $X \oplus Y$ such that the standard projection $P_2(C)$ is compact in Y and let $\mathcal{S} = \{T_\alpha : \alpha \in A\}$ be a family of commuting, uniformly continuous and asymptotically nonexpansive in the intermediate sense self-mappings of C. After establishing demiclosedness principle for such mappings we prove that there exists a nonexpansive $\mathcal{S}$-ergodic retraction $R : C \to \mathrm{Fix}\,\mathcal{S}$ onto the common fixed point set.

Remarks on some recent publications in Fixed Point Theory

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Fixed point theory is one of the areas of the mathematics that attracts considerable attention because of its wide array of applications in not only non-linear analysis but also many branches of sciences. Because of its importance, there is a huge amount of publications on this topic in the literature. But the number of publications in this area is deceiving. Therefore it needs a closer investigation.

In this talk, the claimed view above will be discussed and illustrated with examples.

On Leggett-Williams type theorems for nonlinear operators defined in cones with applications to nonlinear equations

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In the study of nonlinear differential and integral equations the fixed point theorems for completely continuous operators defined in cones play an important role.

Such results often allow not only to prove the existence of a positive solution to a given problem, but also to obtain some additional information on its properties.

During the talk we are going to discuss new Leggett-Williams type theorems, which provide sufficient conditions under which the operator equation of the form $\lambda x = F(x)$, where $\lambda \in \mathbb{R}$ and F is a completely continuous and positive nonlinear operator acting in a partially ordered Banach space, has a positive solution (λ, x) satisfying certain additional conditions. Furthermore, we are going to indicate some applications of our results to the theory of nonlinear differential and integral equations.

A unified approach to iterative construction of common fixed points in nonlinear domains

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W. Takahashi [Kodai Math. Sem. Rep., 22 (1970), 142-149] introduced the notion of convexity in a metric space. Subsequently, many authors have studied fixed point theory in convex metric spaces. Recently, U. Kohlenbach [Trans. Amer. Math. Soc., 357 (2005), 89-128] has enriched convex metric space as hyperbolic space by including some additional conditions. The characterization of a uniformly convex Banach space in terms of an inequality by H. K. Xu [Nonlinear Anal. 16 (1991), 1127–1138, Theorem 2] is playing a pivotal role in the development of the iterative construction of fixed points on linear domains. Y. X. Tian [Comput. Math. Appl. 49(2005), 1905-1912] and I. Yildrim and S. H. Khan [Appl. Math. Comput. 218 (2012), 4860-4866] have considered modified Takahashi's convex structure to study certain iterative algorithms in a convex metric space. We note that neither the modified convex structure contains Takahashi's convex structure as a special case nor it is suitable to generalize the above mentioned characterization of Xu to metric spaces.

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Strong convergence by the shrinking effect of two half spaces

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In this paper, we propose and analyze a new hybrid type shrinking projection method for strong convergence towards a common element of the set of solutions of a finite family of generalized equilibrium problems and the set of common fixed points of two finite families of $k-$ strictly pseudo-contraction maps in Hilbert spaces. Moreover, we characterize such strong convergence by the notion of set convergence.

Approximation of a common fixed point of quasinonexpansive mappings in a geodesic space

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Let us consider the fixed point problem for an operator T on a metric space C, that is, to find a point $z \in C$ satsfying that $z = Tz$. This problem is closely related to various kinds of important problems in nonlinear analysis.

As well as the existence of the solutions, it is also important to study approximation methods for a solution to this problem. We will focus on the shrinking projection method, which was first proposed by Takahashi, Takeuchi, and Kubota [3].

Recently, this kind of approximation method has been generalized to a geodesic space. Kimura [1] considered this method for a family of quasinonexpansive mappings defined on a real Hilbert ball and proved strong convergence of an iterative scheme. Kimura and Satô [2] also obtained a convergence theorem for a single quasinonexpansive mapping on a subset of unit sphere of a Hilbert space.

A mapping T from C into itself is said to be quasinonexpansive if the set of fixed points $F(T) = \{z \in C : z = Tz\}$ is nonempty and

$$d(Tx, z) \leq d(x, z)$$

for every $x \in C$ and $z \in F(T)$. It is obvious that if a nonexpansive mapping has a fixed point, then it is quasinonexpansive.

In this talk, we consider a family of quasinonexpansive mappings defined on a subset of the unit sphere of a real Hilbert space with a spherical metric and obtain convergence of an iterative scheme generated by a shrinking projection method. We also deal with some recent development related to this result.

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A function theoretic generalization of Doss theorem related to Hilbert's 13th problem

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It is famous that Kolmogorov and Arnold solved Hilbert's 13th problem asking if all continuous functions of several real variables can be represented as appropriate superpositions of continuous functions of fewer variables. In 1963, Doss showed a relation between Kolmogorov-Arnold representation theorem and the monotonicity of functions used in this representation theorem. In this talk, we give a nonlinear theoretic generalization of Doss theorem.

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Quantitative aspects of fixed point iterations for Lipschitz pseudocontractive maps

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We present ongoing research concerned with convergence rates of the approximate fixed point sequence for Lipschitz Pseudocontractive maps presented in [1]. We give a highly uniform rate of convergence for the asymptotic regularity, which was extracted in [2] using the methods of [3]. The transition from asymptotic regularity to strong convergence makes use of the fact that a certain convex function, which is definded in terms of a Banach limit, attains its infimum on weakly compact sets. We investigate different possibilities of treating or eliminating Banach limits in order to obtain similar computable bounds for strong convergence in the sense of Tao's metastability.

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Families of k_D-nonexpansive retracts

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We establish a few theorems of Denjoy-Wolff type for some families of k_D-nonexpansive retracts of a bounded and strictly convex domain in complex and reflexive Banach spaces.

Optimal Gronwall Lemmas

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In this paper we present some results relative to concrete Gronwall lemmas which can be derived from abstract Gronwall lemma. Also, we consider some concrete Gronwall lemmas which are not a consequence of the abstract Gronwall lemma, namely, they are not optimal lemmas.

In what follows we consider

Wendorff Lemma. *We assume that:*

(i) $v \in C([0,a] \times [0,b], \mathbb{R}_+)$, $c \in \mathbb{R}_+$;

(ii) v *is increasing.*

If $u \in C([0,a] \times [0,b], \mathbb{R}_+)$ *is a solution of the inequality:*

$$u(x,y) \leq c + \int_0^x \int_0^y v(s,t)u(s,t)dsdt, \ x \in [0,a], \ y \in [0,b], \tag{1}$$

then

$$u(x,y) \leq c \exp\left(\int_0^x \int_0^y v(s,t)dsdt\right). \tag{2}$$

We consider the L-space $(X, \to, \leq) := C(D, \xrightarrow{\|\cdot\|_B}, \leq)$, where $D = [0,a] \times [0,b]$ and $\|\cdot\|_B$ is a Bielecki norm on X.

Let $A : X \to X$ be the operator:

$$A(u)(x,y) = c + \int_0^x \int_0^y v(s,t)u(s,t)dsdt, \ (x,y) \in D. \tag{3}$$

This operator is an increasing Picard operator, but the function

$$(x,y) \to c \exp\left(\int_0^x \int_0^y v(s,t)dsdt\right) \tag{4}$$

is not the fixed point of the operator A and is not a consequence of the abstract Gronwall lemma, therefore is not optimal bound.

In the paper we shall consider some concrete optimal and not optimal Gronwall lemmas in higher dimensions.

A class of P-convex spaces lacking normal structure

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The notion of P-convex space has been introduced by Kottman in [2] as an evaluation of the efficiency of the tightest packing of balls of equal size in the unit ball of X. It has been proved that the condition is weaker at the same time than uniform convexity and uniform smoothness but still implies superreflexivity.

In 2008, the fpp for nonexpansive mappings has been established in the duals of P-convex spaces (F-convex spaces) by Saejung [4] and then in a wider class of dual spaces by Dowling, Randrianantoanina and Turett [1]. It is worth noting that Saejung obtained his result proving that duals of P-convex spaces have uniform normal structure, a property which assures the fpp for nonexpansive maps. Hence the question naturally arises whether P-convex spaces must have uniformly normal or at least normal structure (it is known that these properties are not self-dual).

Here ([3]) I show that this is not true, proving that, for any $\beta > 1$, the space $E_\beta = (l^2, \|\cdot\|_\beta)$, where $\|\cdot\|_\beta = \max\{\|\cdot\|_2, \beta\|\cdot\|_\infty\}$, is P-convex. These spaces have (uniform) normal structure if and only if $\beta < \sqrt{2}$. Therefore the spaces E_β for $\beta \geq \sqrt{2}$ provide, as far as I know, the first examples of P-convex spaces without normal structure.

It has to be remarked that it was proved in literature that all the E_β's do have the fpp for nonexpansive mappings but the problem whether all P-convex spaces enjoy it is still open.

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Asymptotic regularity of firmly nonexpansive mappings

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Because of Minty's classical correspondence between firmly nonexpansive mappings and maximally monotone operators, the notion of a firmly nonexpansive mapping has proven to be of basic importance in fixed point theory, monotone operator theory, and convex optimization. In this note, we show that if finitely many firmly nonexpansive mappings defined on a real Hilbert space are given and each of these mappings is asymptotically regular, which is equivalent to saying that they have or "almost have" fixed points, then the same is true for their composition. This significantly generalizes the result by Bauschke from 2003 for the case of projectors (nearest point mappings). The proof resides in a Hilbert product space and it relies upon the Brezis-Haraux range approximation result. By working in a suitably scaled Hilbert product space, we also establish the asymptotic regularity of convex combinations.

This a joint work with H.H. Bauschke, S.M. Moffat and X. Wang.

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Ulam-Hyers stability of elliptic partial differential equations in Sobolev spaces

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In this work we study the Ulam-Hyers stability of some linear and nonlinear elliptic partial differential equations on bounded open domains of $\mathbb{R}^d, d \geq 2$, with Lipschitz boundary. We use some direct methods and also the theory of Picard operators. In both cases are essential some well-known results such as Sobolev embeddings, Poincaré's inequality and the Cauchy-Schwartz inequality. The main strength of the presented methods is, that we do not have to know the explicit solution of the problem or the corresponding Green functions of the elliptic operators. So the results are general.

One of the obtained results is the Ulam-Hyers stability of the problem

$$\begin{cases} -\sum_{i,j=1}^d \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}(x)\right) + \sum_{i=1}^d b_i(x)\frac{\partial u}{\partial x_i}(x) + (c(x)+\mu)u(x) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $f \in H^{-1}(\Omega), a_{ij}, b_i, c \in L^\infty(\Omega), \forall i, j = 1, \dots, d$, μ is a sufficiently large constant and the a_{ij}'s satisfy a coercivity (or an ellipticity) condition, which is

$$\exists \delta > 0 : \sum_{i,j=1}^d a_{ij}(x)\xi^i\xi^j \geq \delta|\xi|^2,\ \forall \xi = (\xi^1, \dots, \xi^d) \in \mathbb{R}^d \text{ and } \forall x \in \Omega.$$

We also study the nonlinear version of this problem, where f is depending on u.

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Probabilistic contractions with applications in the stability of functional equations

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We compare several classes of probabilistic contractions and discuss some fixed point theorems in connection with the Hyers - Ulam stability of functional equations.

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Relations between classes of multivalued generalized nonexpansive mappings and fixed point theorems

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In recent years, some classes of singlevalued generalized nonexpansive mappings have been introduced and studied. Among others, the so called (L)-type mappings. It is known that Banach spaces with normal structure satisfy the fixed foint property for such (L)-type mappings. Two possible multivalued extensions of this class will be presented, and some fixed point theorems will be given for each one of these two classes.

Some auxiliary mappings generated by families of mappings and solutions of variational inequalities on common fixed points-sets

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Starting by a (finite or infinite) family of mappings, we give many concrete examples of procedures that generate auxiliary mappings. We show that these procedures hide some common properties. These common properties are enough to guarantees the strong convergence of iterative methods that approximate common fixed points that are also the solutions of variational inequality problems involving monotone operators.

On P- and p-convexity of Banach spaces

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We show that every U-space and every Banach space X satisfying $\varepsilon_0(X) < 1$ is P(3)-convex and we study the non-uniform version of P-convexity, which we call p-convexity.

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Graphic contractions

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In this paper we present some considerations about graphic contractions, and we are looking for to give some answers to the following questions:
1. Which generalized contractions are graphic contractions ?
2. Which graphic contractions are weakly Picard operators ?
3. Which graphic contractions are Picard operators ?
Some new proofs for generalized contractions are given, too.

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Some remarks on the bilocal problem

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Using a method proposed by T.A. Burton in [5] and the Contraction Principle, we study the existence, uniqueness and approximation of the solution for a Bilocal Problem. We compare our results with the classical similarly results given by the Fredholm integral operator associated.

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On a functional-differential equation

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In this paper we use fiber contraction principle (see Hirsch W.M.and Pugh C.C., Stable manifolds and hyperbolic sets, *Proc. Symp. in Pure Math.*,**14**(1970), 133-163 and Rus I.A., A fiber generalized contraction theorem and applications, *Mathematica,* Tome **41(64)**, No.1(1999), 85-90), to study the differentiability with respect to delay of the solution of a Cauchy problem for a differential equation with linear modification of the argument.

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On new Furi-Pera type fixed point theorems involving four operators in Banach algebras

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In this paper, we establish new Furi-Pera type fixed point theorems for the sum and the product of four nonlinear operators in Banach algebras; two of the operators are $\mathcal{D}$-Lipschitzian and the other two are completely continuous. Our results extend and improve some existing known ones.

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On a class of nonexpansive-type mappings in geodesic spaces

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In [4], T. Suzuki extended the concept of singlevalued nonexpansive mappings in the following way: a mapping f defined on a subset K of a Banach space is said to satisfy condition (C) if for $x, y \in K$ with $(1/2)\|x - f(x)\| \leq \|x - y\|$, we have $\|f(x) - f(y)\| \leq \|x - y\|$. We further study this condition for both single and multivalued mappings in the setting of hyperconvex and uniformly convex geodesic metric spaces. We present here fixed point, selection and common fixed point results for mappings satisfying condition (C) and related ones.

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On the asymptotic equivalence of a differential system with maxima

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In this paper, one obtain new results on the asymptotic relationship between the solutions of a linear differential system

$$x'(t) = A(t)x(t),\ t \geq a \tag{2}$$

and its perturbed differential system with maxima

$$y'(t) = A(t)y(t) + f(t, y(t), \max_{\xi \in [a,t]} y(\xi)),\ t \geq a. \tag{3}$$

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Fixed point theorems for generalized weak contractions satisfying rational expressions in generalized $g-$cone metric space

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The purpose of this paper is to present some fixed point theorems using generalized weak contractions satisfying rational expressions in generalized $G-$cone metric space which is defined by taking Banach Algebra instead of Banach space.

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Browder's convergence theorem for multivalued mappings without endpoint condition

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In 1967, Browder [1] proved the following theorem.

Theorem. *Let C be a nonempty bounded closed convex subset of a uniformly smooth Banach space X and $t : C \to C$ be nonexpansive. Fix $u \in C$ and define a net $\{x_s\}$ in C by*

$$x_s = (1-s)u + st(x_s),\ \ s \in (0,1).$$

Then $\{x_s\}$ converges strongly as $s \to 1$ to the point of $Fix(t)$ nearest to u.

A natural question arises whether Browder's theorem can be extended to the multivalued case. The first result concerning to this question was proved by Lopez and Xu [2] in 1995. They gave the strong convergence of the net $\{x_s\}$ defined by

$$x_s \in su + (1-s)T(x_s).$$

under the endpoint condition, i.e., $T(y) = \{y\}$ for each $y \in Fix(T)$. Since then the strong convergence of $\{x_s\}$ has been developed and many of papers have appeared. But there is no any result concerning Browder's convergence theorem in linear or nonlinear spaces which completely removes the endpoint condition. In this paper, we prove Browder's theorem for multivalued nonexpansive mappings in a complete $\mathbb{R}-$tree without endpoint condition.

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Fixed points for weakly semicontinuous correspondences and applications in equilibrium theory

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We introduce the notions of weakly semicontinuous correspondences and give a new fixed-point theorem. Several examples are given. As applications we obtain some new equilibrium theorems for abstract economies.

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Another version of the von Neumann-Jordan constant

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We present another version of the von Neumann-Jordan constant and give some of its properties. We also relate it to the fixed point property for k-Lipschitzian and rotative mappings in some Banach spaces, see [1].

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The theory of a fixed point theorem for multivalued operators in b-metric spaces

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Using the weakly Picard operator technique, we will present some fixed point theorems in b-metric spaces, as well as some Ulam-Hyers stability results for operatorial inclusions.

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Fixed point theorems in vector-valued metric spaces with applications

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Let X be a nonempty set. A mapping $d : X \times X \to \mathbb{R}^m$ is called a vector-valued metric on X if it satifies the classical axioms of a metric.

The pair (X, d) is called a generalized metric space if X is a nonempty set and d is a vector-valued metric on X. Notice that the above mentioned concept of generalized metric space is a particular case of a cone metric space (also called K-metric space), with appeared in some works of Krasnoselskii, Kantorovitch, Collatz, Zabreiko, Huang-Zhang, etc.

The classical Banach contraction principle was extended for singlevalued contractions on generalized metric spaces by Perov in [5]. The aim of this talk is to present some fixed point results for multivalued operators in generalized metric spaces in the sense of Perov with applications to operatorial inclusions.

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Sharp evaluation of the spectral radius for mean lipschitzian mappings

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The class of mean lipschitzian mappings was introduced by Goebel, Japón Pineda and Sims in two papers, see [1] and [2]. The standard situation is the following: let (M, ρ) be a metric space, $\alpha = (\alpha_1, \ldots, \alpha_n)$, where $\alpha_i \geq 0, \alpha_1 > 0, \alpha_n > 0$, $\sum_{i=1}^{n} \alpha_i = 1$, we say that $T : M \to M$ is $(\alpha_1, \ldots, \alpha_n)$-lipschitzian for the constant $k > 0$ if for every $x, y \in M$

$$\sum_{i=1}^{n} \alpha_i \rho(T^i x, T^i y) \leq k\rho(x, y).$$

In the case of $n = 1$, we obtain the classical Lipschitz condition: a mapping $T : M \to M$ is called lipschitzian with constant $k > 0$ if for every $x, y \in M$

$$\rho(Tx, Ty) \leq k\rho(x, y).$$

The smallest constant k for which the above inequality holds is called the Lipschitz constant for T and is denoted by $k_\rho(T)$, or simply $k(T)$ when the metric is clear from the context.

Let us define number $k_0(T)$ by

$$k_0(T) = \lim_{m \to \infty} (k(T^m))^{1/m}.$$

In the case of linear mappings, $k_0(T)$ corresponds to the spectral radius of T. In case of nonlinear mappings, we have the following interpretation:

$$k_0(T) = \inf \{k_d(T) : d \text{ is equivalent to } \rho\}.$$

For fixed α and $k > 0$, we give a sharp upper bound for $k_0(T)$, where T is any α-lipschitzian mapping with constant k, see [5].

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Viscosity iteration in CAT(κ)

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We study the approximation of fixed points of nonexpansive mappings in CAT(κ) spaces. We show that the iterative sequence generated by the Moudafi's viscosity type algorithm (compare [1]) converges to one of the fixed points of the nonexpansive mapping depending on the contraction applied in the algorithm. This result is a generalization of [2] and [3] by an application of a new type iterative method.

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Weak Harnack inequalities and multiple positive fixed points

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We present an abstract theory for the existence, localization and multiplicity of fixed points in a cone. The key assumption is the property of the nonlinear operator of satisfying a weak inequality of Harnack type.

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Estimates of the James constant for direct sums and interpolation spaces

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The James constant $J(X)$ plays important role in metric fixed point theory. It is defined by the formula

$$J(X) = \sup\{\min\{\|x+y\|, \|x-y\|\} : x, y \in X,\ \|x\| = \|y\| = 1\}.$$

A Banach space X is uniformly nonsquare if $J(X) < 2$. Given a family of Banach spaces X_s, we show an inequality between James constants of the spaces X_s and the James constant of their direct sum. This yields a characterization of uniform nonsquareness for sums of spaces.

We also show some estimates for the James constant for interpolation spaces obtained via real interpolation method for finite families of spaces. As a corollary we see that if a family contains at least one space which is uniformly nonsquare, then the interpolation space is uniformly nonsquare.

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An application of Schauder's fixed point theorem

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Scalar and matrix Riccati differential equations appear in a number of engineering models, such as the dynamical systems and optimal control. Here, we derive the matrix Riccati equation that corresponds to the mathematical modeling of the optimal control. As an application of Schauder's fixed point theorem, it is proved a matrix Riccati differential equation with continuous periodic coefficients admits at least one periodic solution.

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Fixed point theorems for set-valued maps

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In the last years, several Authors investigated on the existence of fixed points for set-valued maps, because of the applicability to the existence of solutions for integral inclusions or differential inclusion.

Aim of this talk is to give some new results on the existence of fixed points for Monch type set-valued maps in Banach spaces.

This is joint work with Tiziana Cardinali.

Fixed point properties and retractions

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In this talk, we study several fixed point properties for families of nonexpansive mappings in (dual) Banach spaces. In particular, we study the problem of Lau and Mah of whether a dual Banach space has the weak* fixed point property if and only if it has the weak* fixed point property for left reversible semigroups of nonexpansive mappings. We do this through analysis of existence of the nonexpansive retractions [1]. Meanwhile, we discuss the existence of the so-called ergodic retractions over the (common) fixed point sets of some semigroups and families of (asymptotically) nonexpansive mappings.

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A quantitative nonlinear strong ergodic theorem for Hilbert spaces

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In [5], R. Wittmann presented the following strong nonlinear ergodic theorem for (a class of possibly even discontinuous) selfmappings of an arbitrary subset of a

Hilbert space:

Theorem. (Wittmann) *Let S be a subset of a Hilbert space and $T : S \to S$ be a mapping satisfying $\forall u, v \in S$ $(\|Tu + Tv\| \leq \|u + v\|)$. Then for any starting point $x \in S$ the sequence of the Cesàro means $A_n x := \frac{1}{n+1}\sum_{i=0}^{n} T^i x$ is norm convergent.*

In my talk, we investigate the computational content of this theorem. Although in general the sequence of the ergodic averages does not have a computable rate of convergence (even for the von Neumann's mean ergodic theorem for a separable space and computable x and T), as was shown by Avigad, Gerhardy and Towsner in [1], the so called metastable (or quantitative) version nevertheless has a primitive recursive bound. In our case this means that given the assumptions from Wittmann's theorem, the following holds

$$\forall b, l \in \mathbb{N}, g : \mathbb{N} \to \mathbb{N}, x \in S\ \exists m \leq M(l, g, b)\big(\|x\| \leq b \to \|A_m x - A_{m+g(m)}x\| \leq 2^{-l}\big),$$

for a primitive recursive M, which we will define. Note that apart from the counter-function g and the precision l, this bound depends only on b (a bound for the norm of x) and not on S, T, or x itself.

It is one of the goals of this talk to demonstrate that there are proof-theoretic means to systematically obtain such uniform bounds. In fact, for many theorems the existence of a uniform bound is guaranteed by Kohlenbach's metatheorems introduced in [3] and refined in [2]. Additionally, proof theoretic methods such as Kohlenbach's monotone functional interpretation (see [4]) can be used to systematically obtain these effective bounds.

Moreover, we have here a rare example of an application of these techniques to not necessarily continuous operators. In logical terms this amounts to the subtlety that only a weak version of extensionality is available. Also, for the first time, we obtain a bound which in fact makes use of nested iteration.

It is a surprising observation that so far for all metastable versions of strong ergodic theorems primitive recursive bounds could be obtained.

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The Strong and $\triangle$-convergence of S-Iteration Process for generalized nonexpansive mappings on CAT(0) space

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In this paper, we give the strong and $\triangle$-convergence theorems for the S-iteration process of generalized nonexpansive mappings on CAT(0) space which extend and improve many results in the literature.

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Multivalued and singlevalued common fixed point results in partially ordered metric space

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Fixed point theory in partially ordered metric spaces has greatly developed in recent times. In this paper we prove common fixed point results for multivalued and singlevalued mappings in such space. The mappings we consider here are assumed to satisfy certain metric inequalities in the case where the arguments of the functions are related by partial order. Our theorems generalized the results of Rhoades [17], Harjani and Sadarangani [10] and Binayak and Metiya [3].

Fixed points of non-Lipschitzian type mappings in $CAT(0)$ spaces

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The purpose of this talk is to investigate the existence theorem and iterative approximation for fixed points of non-Lipschitzian mappings in the framework of $CAT(0)$ spaces.

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An application of Krasnosel'skii fixed point theorem to nonlinear integral equations

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We present the existence of continuous solutions for a class of nonlinear integral equations in Banach spaces endowed with the weak topology. The result is an application of the generalization of Krasosel'skii fixed point theorem presented by [C Vladimirescu, Remark on Krasnosel'skii fixed point theorem, Nonlinear Analysis 71(2009), 876-880] for locally convex spaces.

Projection algorithms in CAT(0) spaces

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Recently the Alternating Projection Algorithm was extended into $CAT(0)$ spaces [1]. We will discuss further developments in this field. By using $CAT(0)$ spaces the underlying linear structure of the space is dispensable and this allows certain projection algorithms to be extended to spaces such as classical hyperbolic spaces, simply connected Riemannian manifolds of non-positive curvature, R-trees and Euclidean buildings.

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Fibre contraction principle with respect to an iterative algorithm

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Let $(X, +, \mathbb{R})$ be a vectorial space, $Y \subset X$ a convex subset, $f : Y \to Y$ an operator and $\lambda \in]0; 1[$. We denote by

$$f_\lambda : Y \to Y, \quad f_\lambda (x) = (1 - \lambda) x + \lambda f (x) .$$

f_λ is called the Krasnoselskii operator. We have the following result:
Theorem (Fibre contraction principle w.r.t. to Krasnoselskii iteration, I.A. Rus [2]) *Let X and Y be a Banach spaces, $U \subset X$ a convex subset, $V \subset Y$ a closed convex subset, $g : U \to U$, $h : U \times V \to V$ and*

$$f : U \times V \to U \times V, \quad f(u, v) = (g(u), h(u, v))$$

a triangular operator. We suppose that:

(i) g_λ *is WPO for some* $\lambda \in]0; 1[$;

(ii) $(h(u, \cdot))_\lambda : V \to V$ *is* α*-contraction for all* $u \in U$;

(iii) f *is continuous.*

Then:

(a) $F_f = F_{f_\lambda}$;

(b) f_λ *is WPO;*

(c) If g_λ *is PO then* f_λ *is PO.*

In this paper we extend this result to the general case of an *admissible perturbation* for the operator f and some applications are given.

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Fixed point approximations for mappings in geodesic spaces

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In this speech, we study fixed point approximations for mappings in geodesic spaces, and we prove some stability results in fixed point theory for cotraction mappings in geodesic spaces. we also consider some extention theorems for these spaces.

Fixed point theorems and approximation methods for multi-valued mappings

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In this talk, we first discuss the existence of fixed points of multi-valued nonspreading mappings in a Banach space. For the second part of this talk, some approximation methods for multi-valued mappings are discussed. In the last part of the talk, we discuss the rate of convergence of some iterative methods for some nonlinear mappings.

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Recent developments in cyclic contraction

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In this talk, I will talk about the recent developments in cyclic contractions.

KKM theory in modular spaces

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In this talk, we first define the generalized KKM mapping on a modular spaces, and then we apply the property of the modular space to get a characterization of the generalized KKM mapping and the KKM theore

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Asymptotic stability to certain integral equations, via fixed point theory

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In this paper we present existence results of asymptotically stable solutions to certain nonlinear integral equations, by using fixed point theory.

An equilibrium uniqueness result for aggregative games with a constructive proof

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In [1] the following result for the set E of equilibria of a homogeneous Cournot oligopoly game with n firms, inverse demand function $p : \mathbb{R}_+ \to \mathbb{R}$ with market satiation point $v \in]0, +\infty[$, and cost functions $c_i : \mathbb{R}_+ \to \mathbb{R}$ was proven.

Theorem. *Suppose p is decreasing and continuous, $p \restriction [0, v]$ and every c_i is twice continuously differentiable with $Dc_i(x_i) > 0$ $(x_i > 0)$ and for every $y \in [0, v]$ there exists $\alpha < 0$ such that $Dp(y) - D^2c_i \leq \alpha$. Finally,*

$$\sum_{k, e_k > 0} - \frac{D^2p\left(\sum_l e_l\right) e_k + Dp\left(\sum_l e_l\right)}{Dp\left(\sum_l e_l\right) - D^2c_k(e_k)} < 1 \ (\mathbf{e} \in E).$$

Then $\#E \leq 1$. And if $xD^2p(y) + Dp(y) \leq 0$ for every $0 \leq x < v$, then $\#E = 1$.

The difficult part was to prove semi-uniqueness, i.e. that $\#E \leq 1$. Existence, i.e. $\#E \geq 1$, was proved by referring to a Nash equilibrium existence result à la Nikaido-Isoda. The semi-uniqueness proof was based on a refinement of the technique of the cumulative backward best reply correspondence which was developed independently by Selten and Szidarovszky in 1970. In this technique the n-dimensional fixed point problem for the best reply correspondence boils down to a 1-dimensional one.

The theorem concerns a variant of a result in [2]. It improves upon the latter by not excluding degenerate equilibria. The proof given by Gaudet and Salant is much more elementary than the proof of Kolstad and Mathiesen which deals with Cournot equilibria as the solution of a complementarity problem to which differential topological fixed point index theory is applied. Further developing the technique of the cumulative backward best reply correspondence, we not only generalize the above theorem to a class of aggregative games, but also obtain results that substantially improve this theorem intrinsically. In addition we also prove the equilibrium existence part by this technique.

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Minimal displacement and measure of noncompactness

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Let K be a convex closed bounded subset of Banach space X. Denote by $C(K,K)$ the set of all continuous functions from K into K. Define

$$d(K) = sup_{T\epsilon C(K,K)} inf_{x\epsilon K} \|Tx - x\|.$$

We call $d(K)$ the minimal displacement of K. We will try to express $d(K)$ in terms connected with the geometrical structure of K.

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Rational forms that imply the uniqueness and existence of fixed points in partial metric spaces

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By a work of Jaggi, it is known that the existence of certain inequalities for continuous maps over metric spaces implies the existence and uniqueness of fixed points. In this paper, we show that if p denotes a partial metric, the existence of a rational form of type

$$p(Tx,Ty) \leq \frac{\alpha p(x,Tx)\cdot p(y,Ty)}{d(x,y)} + \beta p(x,y)$$

for some α and β with $\alpha+\beta<1$ for a continuous map T over a partial metric space leads to the same conclusions, that is, the existence and uniqueness of fixed points. This is a joint work with Erdal Karapınar.

Generalized hybrid steepest descent method for variational inequality problem over the finite intersection of fixed point sets

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We consider the following variational inequality problem $VIP(\mathcal{F},C)$ in a real Hilbert space $\mathcal{H}$: find $\bar{u}\in C$ such that $\langle \mathcal{F}\bar{u}, z-\bar{u}\rangle \geq 0$, for all $z\in C$, where $\mathcal{F}\colon \mathcal{H}\to\mathcal{H}$ is Lipschitz continuous and strongly monotone, $C:=\bigcap_{i\in I}\operatorname{Fix}U_i\neq\emptyset$ and U_i have the following property: $\langle x-U_ix, z-U_ix\rangle\leq 0$ for all $x\in\mathcal{H}$ and $z\in\operatorname{Fix}U_i$, $i\in I:=\{1,2,...,m\}$. In particular metric projections P_{C_i} onto closed convex subsets $C_i\subseteq\mathcal{H}$ and subgradient projections P_{f_i} for continuous convex function $f_i:\mathcal{H}\to\mathbb{R}$ with $S(f_i,0):=\{x\in\mathcal{H}: f_i(x)\leq 0\}\neq\emptyset$ have this property. It is well known that $VIP(\mathcal{F},C)$ has a unique solution. We propose the following method for $VIP(\mathcal{F},C)$: $u^{k+1}=T_ku^k-\lambda_k\mathcal{F}T_ku^k$, where $T_k:=\operatorname{Id}+\alpha_k(U_{i_k}-\operatorname{Id})$, $\alpha_k\in[0,2]$ and $\{i_k\}\subseteq I$ is a control sequence. If we take $T_k:=T$ for all $k\geq 0$, then we obtain a hybrid steepest descent method (see [1]). We give sufficient conditions for the convergence of sequences generated by the proposed method to the solution of $VIP(\mathcal{F}, C)$. The results show an application of a convergence theorem presented at the same conference by A. Cegielski.

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Non-expanding mappings and fixed points in graph theory

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In this talk we introduce and start studying non-expanding mappings on graphs. We prove the invariance of certain types of subgraphs under such mappings. Then we focus our attention on contractions as peculiar non-expanding mappings and to trees and cacti as special kinds of graphs. The results can be viewed as discrete counterparts of well-known fixed point theorems in arbitrary metric spaces.

Poster Session

New fractional inequalities of Ostrowski type

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In this work we establish a new weighted Montgomery identity for Riemann-Liouville fractional integrals. Then using this new fractional montgomery identity, we obtain some fractional inequalities Ostrowski type.

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On dichotomous evolution operators with unbounded projections

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In this paper we present the notion of uniform dichotomy introduced in [6] and [7] and we point out through an example that, in the hypothesis of uniform dichotomy, the uniform boundedness of the dichotomy projection family fails to be valid by dropping

the assumption of uniform exponential boundedness on the evolution family, although it satisfies a growth property of the type of Barreira-Valls, as introduced in [2].
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Existence theorem for nonlinear perturbed differential equation

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In this paper, a fixed point theorem of Schaefer type involving the sum of two operators in a Banach space is proved and it is further applied to a first order perturbed differential equation for proving the existence theorem under the mixed generalized Lipschitz and Caratheodory conditions.

$$\frac{dx}{dt} = f(t, x(t)) + g(t, x(t)),\ a.e.\ t \in J = [0, 1]$$

$$x(0) = x_0 \text{ and } f, g : J \times C[0, 1] \to C[0, 1].$$

This perturbed differential equation is not new to the literature and has been discussed in the literature since long time. The fixed point of Krasnoselskii is generally used for proving the existence of solution under the mixed Lipshcitzity and Caratheodory conditions. In this paper, we shall also prove the existence theorem for perturbed differential equation using a new nonlinear alternative of Schaefer type to be developed. The general existence principle for first order discontinuous ordinary is also discussed.

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Multiplicity results in Strip-Like Domains

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In the present paper we prove a multiplicity result for a model quasilinear Neumann problem $(\mathcal{N}_\lambda)$, depending on a positive parameter λ,

$$(\mathcal{N}_\lambda)\begin{cases} -\Delta_p u + |u|^{p-2}\cdot u = \beta(x)g(u) & ,x\in\Omega \\ \frac{\partial u}{\partial n} = \lambda\alpha(x)f(u) & ,x\in\partial\Omega. \end{cases}$$

where $\omega \subset \mathbb{R}^m$ with smooth boundary and $\Omega = \omega \times \mathbb{R}^{N-m}$, $p > N$, Δ_p is the p-Laplacian operator, λ is a positive parameter, $\alpha \in L^\infty(\partial\Omega) \cap L^1(\partial\Omega), \beta \in L^1(\Omega)$ are non-zero positive potentials, and $f, g : [0, +\infty[\to \mathbb{R}$ are continuous functions with $f(0) = g(0) = 0$.

By a variational method, we prove that for every $\lambda > \lambda_0$ problem $(\mathcal{N}_\lambda)$ has at least two non-zero solutions, while there exists $\widetilde{\lambda}$ such that problem $(\mathcal{N}_{\widetilde{\lambda}})$ has at least three non-zero solutions.

Common fixed point theorems in Kasahara spaces

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The aim of this paper is to present some common fixed point theorems for self-operators defined on Kasahara spaces $(X, \to, d)$, where $d : X\times X \to \mathbb{R}_+$ is a functional. We extend our results in the context of generalized Kasahara spaces $(X, \to, d)$, where $d : X \times X \to \mathbb{R}^m_+$ is a functional.

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On a D.V. Ionescu's problem for functional-differential equations of second order

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The purpose of this paper is to present the following D.V. Ionescu's problem

$$\begin{cases} -x_1''(t) = f_1(t, x_1(t), x_2(t), x_1'(t), x_2'(t)), \ t \in [a,b] \\ -x_2''(t) = f_2(t, x_1(t), x_2(t), x_1'(t), x_2'(t)) \end{cases} \tag{4}$$

with polylocal conditions

$$\begin{cases} x_1(a) = x_2(b) = 0 \\ x_1(c) = x_2(c) \\ x_1'(c) = x_2'(c). \end{cases} \tag{5}$$

Existence, uniqueness and data dependence (continuity, differentiability with respect to parameter) results of solution for the Cauchy problem are obtained using Perov fixed point theorem and weakly Picard operator theory.

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Fixed point theorems for multivalued contractive operators on generalized metric spaces

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Using the concept of generalized w-distance we prove some interesting results on the existence of fixed points for multivalued contractive type operators in the setting of generalizes metric spaces. A data dependence result and Ulam-Hyers stability of fixed point inclusions are also presented.

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On a quasilinear and singular non-cooperative elliptic system

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In this paper, we study the following quasilinear and singular problem

$$\begin{cases} -\Delta_{p_1} u = u^{\alpha_1} v^{\beta_1} \text{ in } \Omega, \\ -\Delta_{p_2} v = u^{\alpha_2} v^{\beta_2} \text{ in } \Omega, \\ u, v = 0 \quad \text{ on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain with smooth boundary and the exponents α_i and β_i $(i = 1, 2)$ are real numbers such that $\alpha_1, \beta_2 > 0$ and $\min(\alpha_2, \beta_1) < 0$. Under suitable conditions on the exponents $\alpha_1, \alpha_2, \beta_1$ and β_2, an existence result is proved by using Schauder's fixed point theorem. The uniqueness of the solution is also established when the exponents β_1 and α_2 are both negative.

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Existence and Ulam-Hyers stability results for coincidence problems

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In this paper, we will present some existence and Ulam-Hyers stability results for fixed point and coincidence point problems for singlevalued and multivalued operators in metric spaces. Our approach is based on Picard and weakly Picard operator technique. Some examples illustrating the main results of the paper are also given.

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A fixed point approach to nonlocal initial value problems for first order differential systems

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The purpose of the present paper is to study the existence of solutions to initial value problems for nonlinear first order differential systems with nonlocal conditions. The proof will rely on the Perov, Schauder and Leray-Schauder fixed point principles which are applied to a nonlinear integral operator. The novelty in this paper is that this approach is combined with the technique that uses convergent to zero matrices and vector norms.

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On the behavior of the solution to a contact problem with memory term

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We consider a quasistatic contact problem between a viscoplastic body and an obstacle, the so-called foundation. The contact is frictionless and is modelled with a new and nonstandard condition which involves both normal compliance, unilateral constraint and memory effects. The unique weak solvability can be proved using a fixed point result. In the present paper we analyse the dependence of the solution of this problem with respect to the data. We state and prove a convergence result and present numerical simulations.

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Krasnoselskii's Theorem in E-Banach spaces and applications

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In this paper we present an extension of a fixed point principle given in [3] for singlevalued and multivalued operators which satisfies nonlinear φ-contraction conditions in E-metric spaces. We also extend Krasnoselskii's fixed point theorem for the sum of two operators to the case of E-Banach spaces. An application to a Fredholm-Volterra type integral equation and inclusion is also given.

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Best proximity point theorems for weak p-cyclic Kannan contractions

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We introduce the notion of weak p-cyclic Kannan contraction. We give a lemma which shows that the distance between the adjacent sets are equal under weak p-cyclic Kannan contraction. Also, we obtain some convergence and existence results for best proximity points for weak p-cyclic Kannan contractions in the setting of a uniformly convex Banach space.

Generalized variational inclusions with H(., ., .)-cocoercive mixed operator

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In this paper, we introduce and study a new system for generalized variational inclusion in Hilbert spaces and define an iterative algorithm for finding the approximate solutions of the class of system of variational inclusions with the help of $\mathbf{H}(.,.,.)-$ cocorecive mixed operator. We also establish that the approximate solutions obtained by our algorithm converges to the exact solution of new system of generalized variation inclusions for solving various known equilibrium problem and other related problem.

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Fixed points theorems for multivalued weakly Reich-contractive operators

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In this paper we recall the notion of w-distance and we define the concept of weakly Reich-contractive operator. Then we give a fixed point result for this type of operators and we study the data dependence for this new result.

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Some contributions to the theory of coupled fixed points

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This paper deals with some coupled fixed points results for contractive type singl-evalued and respectively multivalued operators on spaces endowed with vector-valued metrics. The case of mixed monotone operators is also considered. Some applications to integral equations and to boundary value problems are given.

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Fixed set theorems with application to the fixed point theory

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Here, we show that continuous set-valued maps which are generalized set contraction on noncompact topological spaces have a maximal invariant (fixed) set. As an application, we prove the existence and uniqueness of endpoints for topological contraction mappings. Also, we present fractal set results for system of continuous set-valued maps on regular topological spaces. As application of our result, we show how some fixed point theorems can be established from these results.